

Why is the Moon synchronously rotating?

Valeri V. Makarov[★]

US Naval Observatory, 3450 Massachusetts Ave NW, Washington, DC 20392-5420, USA

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ABSTRACT

If the Moon's spin evolved from faster prograde rates, it could have been captured into a higher spin–orbit resonance than the current 1:1 resonance. At the current value of orbital eccentricity, the probability of capture into the 3:2 resonance is as high as 0.6, but it strongly depends on the temperature and average viscosity of the Moon's interior. A warmer, less viscous Moon on a higher eccentricity orbit is even more easily captured into supersynchronous resonances. We discuss two likely scenarios for the present spin–orbit state: a cold Moon on a low-eccentricity orbit and a retrograde initial rotation.

Key words: Moon – planets and satellites: dynamical evolution and stability.

1 INTRODUCTION

The origin of the Moon and the circumstances of its dynamical evolution remain unclear to date, despite the numerous studies on this issue. The low eccentricity ($e = 0.0549$) and the exactly synchronous rotation suggest a nearly perfect equilibrium state, which is the end-point of dynamical spin–orbit evolution (Hut 1980). They also point at a protracted history of dynamical interaction with the Earth and the Sun, in which tidal dissipation undoubtedly played a crucial role. In the framework of the giant impact theory of Moon's origin, tidal dissipation is responsible for the expansion of the orbit and damping of eccentricity (Ćuk & Stewart 2012). Numerical simulations validating this hypothesis have been based on much simplified and ad hoc models of tides, which should not be used for planets and moons of terrestrial composition (Efroimsky & Makarov 2013). In recent years, a more realistic model of tidal dissipation in solid bodies was proposed, which combines the viscoelastic response with the inelastic creep (Efroimsky & Lainey 2007; Efroimsky & Williams 2009; Efroimsky 2012). In the framework of this model, the capture of Mercury into the current 3:2 spin–orbit resonance becomes a likely and natural outcome even without involving the core–mantle friction and episodes of high orbital eccentricity (Makarov 2012), which has been a difficult issue for the previous theories. For example, the constant time lag (CTL) model predicts capture probabilities into 3:2 of less than 0.1 for a wide range of parameters (Goldreich & Peale 1966). Within the Efroimsky (2012) model, the secular tidal torque is rendered by a Darwin–Kaula expansion over the Fourier modes of the tide. Each term of the series assumes, in the vicinity of the appropriate resonance, the shape of a kink. As the tidal mode corresponding to the term transcends zero, the term swiftly, but continuously, changes its sign vanishing at the resonance frequency. This behaviour makes the Efroimsky torque very efficient at trapping the spin rate into

resonances of higher order than the synchronous rotation. Assuming a terrestrial composition for the super-Earth GJ 581d, Makarov, Berghia & Efroimsky (2012) concluded that this potentially habitable exoplanet is more likely to be found at a 2:1 spin–orbit resonance rather than 1:1. In this Letter, I reassess the probabilities of capture of the Moon into supersynchronous resonances.

2 SECULAR TIDAL TORQUE

Comprehensive equations for the polar tidal torque (i.e. the component directed along the axis of rotation), including the fast oscillating terms, can be found, for example, in Makarov et al. (2012). They are not reproduced here for brevity. The secular term of the torque, K_c , is strongly dependent on tidal frequency in the narrow vicinity of spin–orbit resonances $(2 + q)n = 2\dot{\theta}$ for integer q , where n is the mean motion and $\dot{\theta}$ is the sidereal spin rate. The characteristic kink shape of the near-resonant torque is present at both 1:1 and 3:2 resonances of the Moon, but the former is by far larger than the latter and the other higher order resonances (Fig. 1). The torque is positive, or accelerating, at forcing frequencies below the resonance value and negative, or decelerating, above it. The very steep decline between the two peaks occupies a narrow band of frequencies for realistic rheologies. Despite the relatively small amplitude of the kink (compared to a typical amplitude of the triaxiality-caused torque), it acts as an efficient trap for a planet trying to traverse the resonance.

The numerical simulations presented in this Letter were performed with physical parameters listed in Table 1. The Andrade parameter α has been measured for a diverse list of materials, including silicates, metals and ices, and found to vary within a fairly narrow range of 0.14–0.3. The value 0.2 estimated for hot silicate rocks is used in this Letter. The unrelaxed rigidity modulus μ takes values between 0.62×10^{11} and 0.68×10^{11} Pa (Eckhard 1993). The assumed value here is 0.65×10^{11} Pa. The most defining parameter in this model is the Maxwell time τ_M , which I varied in my analysis between 8 and 500 yr (approximately, the Earth's value). The former value corresponds to a warmer satellite with less

[★]E-mail: vvm@usno.navy.mil

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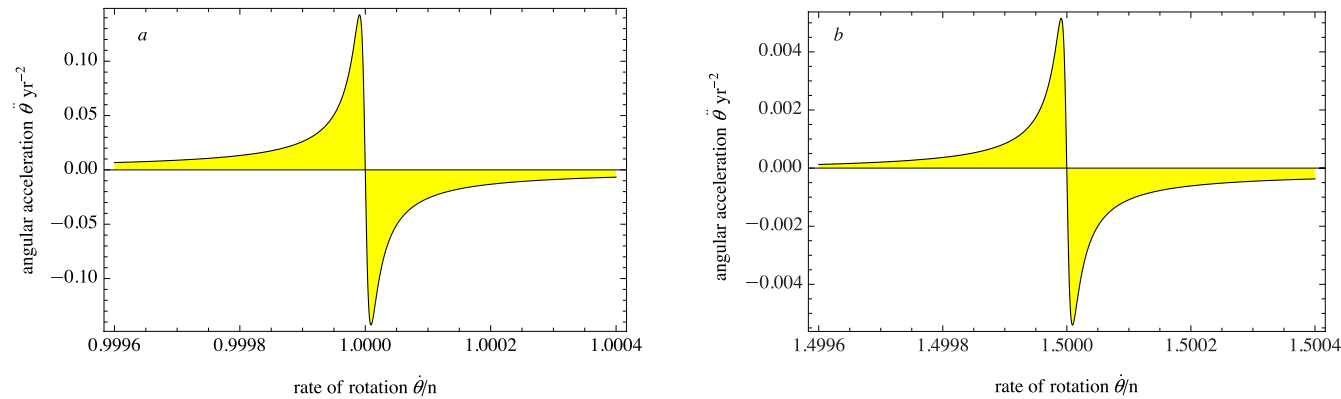


Figure 1. Angular acceleration of the Moon caused by the secular component of the polar tidal torque in the vicinity of (a) 1:1 resonance and (b) 3:2 resonance.

Table 1. Parameters of the tidal model.

Name	Description	Units	Values
ξ	Moment of inertia coefficient		2/5
R	Radius of planet	m	1.737×10^6
M_2	Mass of the perturbed body (Moon)	kg	7.3477×10^{22}
M_1	Mass of the perturbing body (Earth)	kg	5.97×10^{24}
a	Semimajor axis	m	$3.843\,99 \times 10^8$
n	Mean motion, i.e. $2\pi/P_{\text{orb}}$	yr ⁻¹	84
e	Orbital eccentricity		0.0549
$(B - A)/C$	Triaxiality		2.278×10^{-4}
G	Gravitational constant	m ³ kg ⁻¹ yr ⁻²	664 68
τ_M	Maxwell time (Ratio of viscosity to unrelaxed rigidity)	yr	8
μ	Unrelaxed rigidity modulus	Pa	0.8×10^{11}
α	The Andrade parameter		0.2

internal viscosity. As explained in Makarov & Efroimsky (2013), the choice of a small Maxwell time for the Moon, only 8 yr, may be justified by the likely presence of a high percentage of partial melt in the lower lunar mantle. The presence of partial melt indirectly follows from the modelling carried out by Weber et al. (2012) and also from an earlier study by Nakamura et al. (1974). Fig. 2(a) shows a typical example of numerical integration of the Moon’s spin rate, which includes both secular and oscillating components of tides raised by the Earth, as well as the triaxiality-induced torques. The initial rate is $\dot{\theta} = 1.572n$ and the maximum step of integration is 1.5×10^{-4} yr. The plot shows the characteristic features of resonance capture: the spin rate decelerates steadily and at a nearly

constant rate on this time-scale, the amplitude of free librations grows towards the resonance and suddenly doubles upon the capture, after which it starts to decline due to the dissipation of kinetic energy. More remarkable is the fact that the Moon is captured into the 3:2 resonance, despite its nearly circular orbit. Thus, capture of the Moon into supersynchronous resonances is possible with the present-day parameters.

3 PROBABILITIES OF CAPTURE

There are two ways of estimating the probability of capture into a spin–orbit resonance with a given set of parameters. The first way is

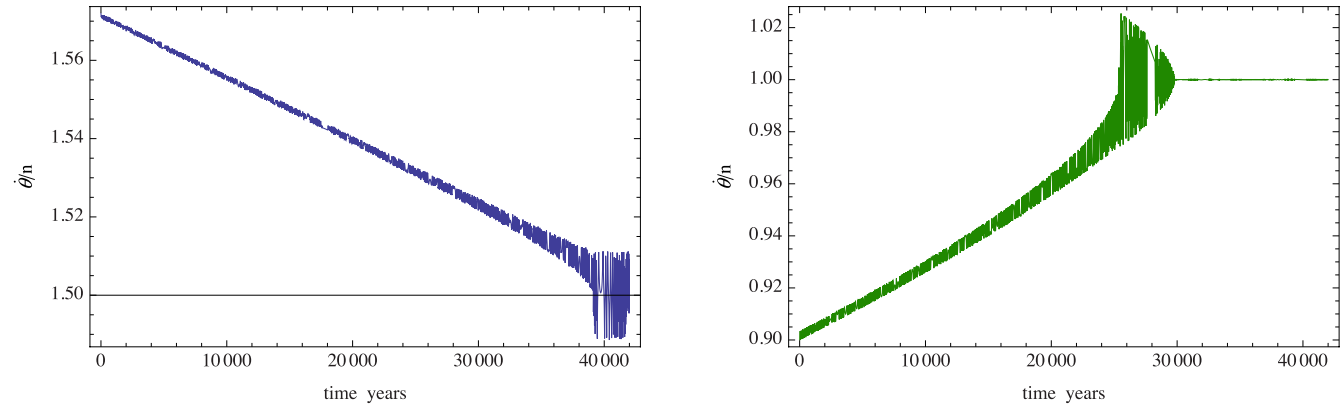


Figure 2. Capture of the Moon in numerical simulations into (a) 3:2 spin–orbit resonance from faster prograde rotation and (b) 1:1 resonance from initially slower prograde rotation.

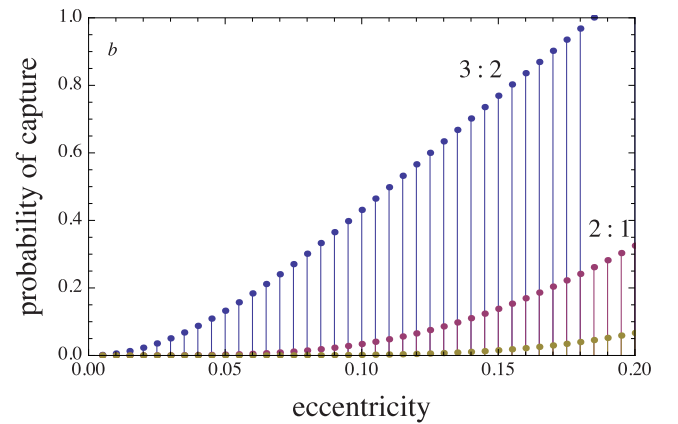
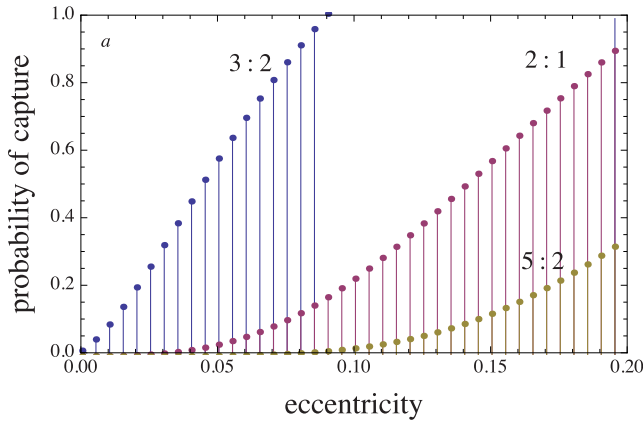


Figure 3. Probabilities of capture of the Moon into the 3:2, 2:1 and 5:2 spin-orbit resonances as functions of eccentricity for (a) $\tau_M = 8$ yr and (b) $\tau_M = 500$ yr.

brute-force integrations of the differential equation of second order for the angular acceleration caused by the polar component of the tidal torque acting on the Moon and for a grid of initial phase space parameters $\{\theta, \dot{\theta}\}$. For the sake of simplicity, but without a loss of generality, these integrations are started at zero mean anomaly, $\mathcal{M}(0) = 0$, i.e. at perigees. The implicit assumption used in this method is that any sidereal azimuthal angle θ is equally likely for a given spin rate $\dot{\theta}$ when the Moon passes through a perigee. I performed small-scale simulations, integrating the corresponding second-order ODE 20 times for these initial parameters: $\dot{\theta}(0) = 1.572 n$, $\mathcal{M}(0) = 0$, $\theta(0) = (j - 1)\pi/20$ and $j = 1, 2, \dots, 20$, the Maxwell time being fixed at 8 yr. I found 12 captures and 8 passages, resulting in a capture probability of roughly 0.6.

The other way of estimating capture probabilities is the adaptation of the derivation proposed by Goldreich & Peale (1968) for the constant phase lag and the CTL tidal models. The details of this calculation are given in Makarov (2012) and, in greater detail, in Makarov et al. (2012). Figs 3(a) and (b) depict the results for two characteristic values of τ_M , 8 and 500 yr, respectively. The results also depend on the measure of quadrupole elongation, $(B - A)/C$, but to a lesser degree. It should be noted that this semi-analytical calculation is based on the assumption that the energy offset from zero at the beginning of the last libration above the resonance is uniformly distributed between 0 and the total energy dissipated by the secular tidal torque along the separatrix trajectory during one free libration cycle (see, e.g., Peale 2005). This assumption is probably quite good as long as the magnitude of the permanent figure's torque is much greater than the magnitude of tidal torques. Caution should be exercised with this approach for nearly axially symmetrical bodies, which are more easily captured into spin-orbit resonances, all other parameters being the same. The strong non-linearity of the tidal force may skew the probability distribution of the residual rotational energy at the beginning of the last pre-resonance libration. Given this caveat, we confirm that the capture probabilities strongly depend on the value of τ_M . For example, as shown in Fig. 3, the probability of capture into 3:2 is 0.58 for $\tau_M = 8$ yr and 0.16 for $\tau_M = 500$ yr. At first glance, these numbers may seem to be consistent with the current state of Moon's rotation, as the probability of traversing the higher resonances and entrapment in the 1:1 resonance (which is always certain) is at least ~ 0.4 for a wide range of the least known parameter τ_M . However, recall that these estimates are obtained with the current low eccentricity. Why the high probabilities of capture into a supersynchronous rotation represents a hard theoretical problem will be discussed in Section 5.

4 SPIN-DOWN TIMES

The secular tidal torque in the model under investigation is negative at the present-day eccentricity for any $\dot{\theta} > n$ except for close vicinities of a few low-order spin-orbit resonances. Therefore, the general action of the tides raised by the Earth on the Moon is to slow down the prograde rotation of the latter. Of special importance is the characteristic spin-down time, which, following the previous literature (e.g. Rasio et al. 1996), can be defined as

$$t_\theta = \frac{\dot{\theta}}{|\ddot{\theta}_{(T)}(\dot{\theta})|} \quad (1)$$

with $\ddot{\theta}_{(T)}(\dot{\theta})$ being the angular acceleration caused by the secular tidal torque $\langle T \rangle$. In this computation, as ever, the obliquity of the lunar equator is ignored. The results are shown in Fig. 4 for a grid of points in $\dot{\theta}/n$, chosen in such a way so as to avoid the sharp features at spin-orbit resonances, for two values of eccentricity: $e = 0.054$ and 0.3 , and for the current value of semimajor axis, which is about 60 Earth's radii.

Let us recall that in the 'work-horse' tidal model of CTL, the deceleration of spin is arrested when the state of pseudo-synchronous rotation is reached at $\dot{\theta}_{\text{pseudo}}/n \approx 1 + 6e^2$ (e.g. Hut 1980). In reality, pseudo-synchronous equilibria are unstable for terrestrial planets and moons (Makarov & Efroimsky 2013). Therefore, at small or moderate eccentricities, the Moon is bound to spin-down

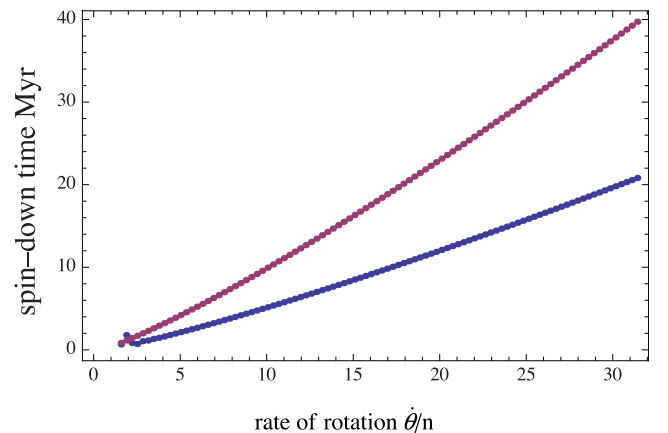


Figure 4. Characteristic times of spin-down of the Moon for the current value of orbital eccentricity $e = 0.055$ (upper curve) and $e = 0.3$ (lower curve). In both cases, the current value of semimajor axis is assumed and τ_M is set at 8 yr.

continuously until it is captured into one of the spin–orbit resonances. Furthermore, most of the theories of Moon’s origin suggest that the Moon was formed much closer to the Earth than it is now (Canup 2004). The characteristic spin-down times are strong functions of the semimajor axis through the relation to the polar torque, $\langle T \rangle \propto a^{-6} K_c(\dot{\theta}, n)$, where K_c is the frequency-dependent quality function defined in Efroimsky (2012) and Makarov (2012). For example, if we compute the characteristic times for the same relative rates $\dot{\theta}/n$ and $a = 8R_{\text{Earth}}$, we obtain practically the same curves as in Fig. 4, but scaled down by approximately 5000. Observe that the dependence of tidal dissipation on a is much weaker here than in the CTL model, which predicts $t_{\theta} \propto a^6 \dot{\theta}/n$ (Goldreich & Peale 1968; Correia & Laskar 2009), due to the fast decline of the quality function K_c with tidal frequency.

5 DISCUSSION

One of the theoretical difficulties that the currently dominating giant impact theory of lunar formation encounters is the excess angular momentum of the early Earth–Moon system. Čuk & Stewart (2012) suggested a dynamical scenario, which allows a fast-spinning proto-Earth to lose a sufficiently large amount of angular momentum after a debris disc forming impact through a relatively short epoch of capture into the evection resonance. As first suggested by Yoder in 1976, according to Peale & Cassen (1978), and mathematically developed by Touma & Wisdom (1998), the lunar perigee is locked in a synchronous precession with the orbital motion of the Earth around the Sun, and the long axis of the lunar orbit stays at 90° from the Sun–Earth line. This resonance defeats the tidal actions of circularization (secular decrease of eccentricity) and orbital expansion, allowing the eccentricity to remain high for a span of time sufficiently long for the Earth to spin down. In the numerical simulation presented by Čuk & Stewart (2012), the evection resonance holds for approximately 60 Kyr. Unfortunately, the authors used a variant of the ‘constant- Q ’ model, which is not adequate for solid or partially melted bodies (Efroimsky & Makarov 2013). Their conclusions about the early dynamical evolution of the Moon–Earth system should be taken with a grain of salt. The main difference between this ad hoc model and the realistic rheological model is that in the latter, the quality function is a rising function at positive tidal frequencies asymptotically approaching zero (Fig. 1). The weakening of tidal dissipation at high tidal frequencies may resolve the problem of overheating the Moon, as discussed in Section 4. The spin-down of the early Moon is still fast enough (a few Kyr) to justify the widely accepted assumption in numerical simulations that the Moon is already synchronized by the time the evection resonance sets in. So much the more puzzling becomes the issue how the Moon traversed the higher spin–orbit resonances on its way to synchronous rotation.

Indeed, capture into the 3:2 resonance becomes certain at $e \simeq 0.09$ for $\tau_M = 8$ yr and $e \simeq 0.18$ for $\tau_M = 500$ yr (Fig. 3). The simulations by Čuk & Stewart (2012) suggest that the orbital eccentricity acquires much higher values shortly after the onset of the evection resonance. Furthermore, these probabilities are computed for the current mean motion of the Moon, whereas the giant impact theory implies much smaller orbits for the early Moon, down to $4R_{\text{Earth}}$. Somewhat counter-intuitively, the probabilities of capture into a spin–orbit resonance become smaller for tighter orbits, all other parameters being the same. For example, the probability of capture into 3:2 is only 0.2 for the Moon at $a = 4R_{\text{Earth}}$, $\tau_M = 8$ yr and $e = 0.055$. Could the Moon traverse the 3:2 resonance (and all the higher resonances) while it was still very close to the Earth? Our

calculations show that the Moon is inevitably entrapped in the 3:2 resonance at $a = 4R_{\text{Earth}}$, if the eccentricity exceeds 0.17. But the evection mechanism quickly boosts the orbital eccentricity to much higher values, up to $\simeq 0.6$. Therefore, the only realistic possibility for the Moon to avoid the 3:2 resonance within the giant impact scenario is to spin down to its present-day synchronous state before the onset of the perigee precession resonance. This may take, depending on the initial spin rate, up to 10 Kyr. This scenario also requires that the Moon remains fairly cold and viscous during this pre-evection stage, which, due to the proximity to the Earth, may prove another hard problem.

Simple calculations based on the formulae in Peale & Cassen (1978) show that the dissipation of tidal energy in the Moon may exceed 10^{23} J yr $^{-1}$ for $a = 10R_{\text{Earth}}$, $\tau_M = 8$ yr and $e = 0.055$ in the vicinity of the 3:2 resonance. This may raise the temperature of the Moon by $\simeq 1$ K in 1 Kyr. The rise of temperature may be much faster at smaller distances from the Earth because of the implicit $dE/dt \propto a^{-15/2}$ relation. For this calculation, I updated equation (31) in Peale & Cassen (1978) by including the realistic frequency-dependent quality function $K_c(\chi_{lmpq})$ instead of the constant quality factor $\frac{3}{5}h_2/Q_{lmpq}$ used in that paper, and inserting the actual frequency mode. The latter update takes into account that the original equation was derived specifically for the synchronous resonance. The resulting general equation is

$$\left\langle \frac{dE}{dt} \right\rangle = \frac{GM_1^2 R^5}{a^6} \sum_{m=0}^2 \frac{(2-m)!}{(2+m)!} (2 - \delta_{0m}) \times \sum_{p=0}^2 \sum_{q=-\infty}^{+\infty} [F_{2mp}(i) G_{2pq}(e)]^2 \chi_{2mpq} K_c(\chi_{2mpq}), \quad (2)$$

where G is the gravitational constant, M_1 is the mass of the Earth, R is the radius of the Moon, a is the semimajor axis of the orbit, i is the Moon’s equator obliquity, $F_{2mp}(i)$ are the inclination functions, $G_{2pq}(e)$ are the eccentricity functions, $\chi_{2mpq} = |\omega_{2mpq}| = |(2 - 2p + q)n - m\dot{\theta}|$ is the tidal frequency and n is the orbital mean motion. This estimation is limited to the leading degree $l = 2$ because the higher degree terms are smaller in amplitude by at least several orders of magnitude. The specific equations for the quality function can be found in Efroimsky (2012). One of the essential differences between equations (2) and equation 31 in Peale & Cassen (1978) is the positively defined tidal frequency χ_{2mpq} in the former replacing the factor $(2 - 2p + q - m)n$ in the latter, which can change sign. An accurate derivation of the dE/dt equation shows that the rate of tidal dissipation is proportional to $\omega_{lmpq} k_l(\omega_{lmpq}) \sin \epsilon_l(\omega_{lmpq})$, where k_l is the frequency-dependent dynamical Love number and ϵ_l is the degree- l phase lag. Observing that k_l is an even function of the tidal mode and ϵ_l is an odd function, this product can be more concisely written as the positively defined function $\chi_{2mpq} K_c(\chi_{2mpq})$ of the physical frequency. The resulting rate of tidal heating from equation (2) may therefore be significantly higher than the previously published estimates. The leading terms of the quality function $K_c(\chi_{2mpq})$ vanish at the corresponding tidal modes, for example, $K_c(\chi_{2200}) = 0$ for $\dot{\theta} = 1n$, turning to zero the tidal torque and acceleration. That does not, however, imply that tidal dissipation almost ceases when the planet is locked in a spin–orbit resonance. The presence of other $lmpq$ modes, multiplied by their tidal frequencies, makes up for a significant net dissipation. The character of the tidal heating versus spin rate dependence is distinctly different with this model, to be discussed elsewhere.

Peale & Cassen (1978) briefly mention the possibility that the Moon was locked into the 3:2 spin–orbit resonance for a finite time

span. A similar suggestion was made by Garrick-Bethell, Wisdom & Zuber (2006), who also found evidence for a high-eccentricity episode in the dynamical history of the Moon from its present-day shape. Capture into a spin-orbit resonance should have happened before or at the very beginning of the evection resonance, while the distance to the Earth remained small. If the subsequent rise of eccentricity finds the Moon still in the 3:2 spin-orbit resonance, the tidal dissipation becomes a few orders of magnitude stronger and a complete or partial melt-down may follow. For example, the dissipation for $a = 10 R_{\text{Earth}}$, $\tau_M = 8 \text{ yr}$ and $e = 0.5$ is $\sim 10^{24.5} \text{ J yr}^{-1}$. This would be sufficient to heat the Moon by 3.6 K per century. If the epoch of high eccentricity during the evection resonance lasts for 40 Kyr, the temperature rises by $\sim 1440 \text{ K}$, which is above the melting point of silicates, including olivine and pyroxene. Besides, it is not obvious what kind of dynamical action could drive the Moon out of the resonance, apart from a fortuitous high-velocity impact from an external body. Once captured into a spin-orbit resonance, a triaxial body can traverse it only through a small opening in the phase space (Makarov 2012). In particular, the angle between the 'long' axis of the body and the centre line should reach nearly 90° at perigee for this to happen. Upon capture into the 3:2 resonance, the amplitude of the angle variation is close to that threshold value, but the lunar free librations are damped quickly because of the high tidal dissipation, and the forced librations are usually insignificant. Beyond the evection resonance, the eccentricity is bound to decrease, further reducing the amplitude of forced librations.

Outside of the giant impact hypothesis of lunar origin, other plausible scenarios exist, which are consistent with the current state of the satellite. If the Moon always remained on a low-eccentricity orbit during the initial spin-down epoch, and it was cold and unyielding to the tidal forces, it could naturally traverse the supersynchronous resonances before settling in the 1:1 resonance. Alternatively, the Moon could have a retrograde rotation at its formation. The tidal pull of the Earth in this case will slow down the retrograde spin, and

then will spin the Moon up in the prograde direction, until it falls into the 1:1 resonance, as shown in Fig. 1(b). The only obstacle on this way is the subsynchronous 1:2 resonance. This resonance, however, is significantly weaker than the 1:1 and 3:2 resonances, and an unhindered passage is secured with not too high eccentricities.

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